UNIT - VIII

ATOMS and Nuclei

Q.1. How is the impact parameter related to angle of scattering? **1.**

Ans. Impact parameter b,

$$\boxed{b = \frac{1}{4\pi\epsilon_0} \frac{Ze^2 \cot\frac{\theta}{2}}{\frac{1}{2}mv^2}}$$

Q.2. What is the relation for Paschan series line of hydrogen spectrum? **1**

Ans: $\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$, $n = 4, 5, 6, ---$

Q.3. What is the ratio of the radii of two nuclei of mass numbers A_1 and A_2.? **1.**

Ans. $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$.

Q.4. Write the nuclear decay process for β decay of ${}_{15}P^{32}$. **1.**

Ans. ${}_{15}P^{32} \longrightarrow {}_{16}S^{32} + \bar{e} + \bar{\nu}$.

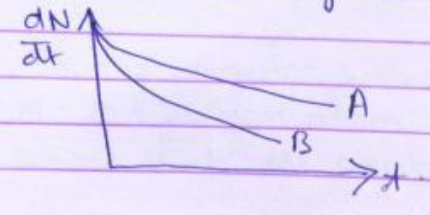
Q.5. What is nuclear holocaust? **1.**

Ans. Nuclear holocaust is the name given to the large scale destruction and devastation that would be caused by the use of nuclear weapons.

Q.6. A nucleus ${}_{n}X^{m}$ emits one α- particle and one β particle. What is the mass number and atomic number of the product nucleus? **1.**

Ans: ${}_{n}X^{m} \xrightarrow{\alpha} {}_{n-2}Y^{m-4} + {}_{2}He^{4} \xrightarrow{{}_{-1}\beta^{0}} {}_{n-1}Z^{m-4} + {}_{-1}\beta^{0}$

Q.7. Which nucleus has greater mean life, A or B? **1.**



Ans: For B. mean life (τ) is more.

As, $R = R_0 e^{-\lambda t}$

slope of graph $= \frac{dR}{dt} = -\lambda R$

Slope of A > slope of B

$[\lambda]_A > [\lambda]_B$ But $\tau \propto \frac{1}{\lambda}$

$[\tau]_B > [\tau]_A$.

Q.8: Explain whether the neutron proton ratio increases or decreases during β-decay. **1.**

Ans: The ratio decreases.

In each β-decay, the no. of neutron decreases by one and the no. of proton increases by 1.

Q.9. Find the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atom? **1.**

Ans:

$$r_n \propto n^2$$

$$\frac{r_2}{r_1} = \frac{2^2}{1^2} = \frac{4}{1} = 4$$

Q.10. What is angular momentum of an electron in 3rd orbit of an atom? **1.**

Sol. ~~From~~ from Bohr's II postulate

$$L = mvr = n\frac{h}{2\pi} \quad \text{here } n=3$$

$$L = 3 \times \frac{6.63 \times 10^{-34} \times 7}{2 \times 22}$$

$$= \frac{3 \times 46.41 \times 10^{-34}}{2 \times 22} = \frac{3 \times 23.2 \times 10^{-34}}{22}$$

$$= \frac{3 \times 11.5 \times 10^{-34}}{11} \simeq \boxed{3.2 \times 10^{-34}\, Js.}$$

— 2 marks, Q —

Q.11. What is the angular momentum of an electron in Bohr's hydrogen atom whose energy is $-3.4eV$?

Ans: We know that the energy of an electron in

n^{th} Bohr orbit of hydrogen atom is given by

$$E = -\frac{13.6}{n^2} \text{ eV}$$

$$-3.4 = -\frac{13.6}{n^2}$$

$$n^2 = 4 \Rightarrow \boxed{n = 2}$$

Angular momentum of an electron in n^{th} orbit

$$L = \frac{nh}{2\pi}$$

$$L = \frac{2h}{2\pi} = \frac{h}{\cancel{2}\pi}$$

$$\boxed{L = \frac{h}{\pi}} = \frac{6.63\times10^{-34}}{3.14} = \boxed{2.11\times10^{-34} \text{ J.s}}$$

Q.12. Show that the instantaneous rate of change of activity of a radio-active substance is inversely proportional to square of its half life.

Ans: We know that,

$$R = R_0 e^{-\lambda t}$$

differentiating both sides w.r.t. 't',

$$\frac{dR}{dt} = -\lambda R_0 e^{-\lambda t}$$

$$= -\lambda R$$

$$= -\lambda(\lambda N)$$

$$\frac{dR}{dt} = -\lambda^2 N$$

*Actually
$N = N_0 e^{-\lambda t}$
Rate of decay
$R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$
$R = \lambda N$

But $\lambda = \frac{0.6931}{T}$, then

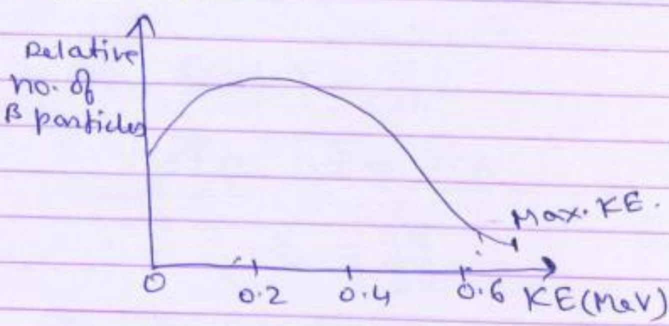
$$\frac{dR}{dt} = -\left(\frac{0.6931}{T}\right)^2 N$$

$$\boxed{\frac{dR}{dt} \propto \frac{1}{T^2}}$$

Q.13: What is the ratio of nuclear densities of two nuclei which mass numbers in the ratio 1:3?

Also plot the distribution of KE of β particles and why the energy spectrum is continuous:

Ans: Since the nuclear density is independent on mass number, so ratio of nuclear densities will be 1:1.



The energy available in β particle is shared in all possible proportions by the electron and antineutrino. So the energy spectrum of β ray is continuous.

Q.14. The ground state energy of hydrogen atom is −13.6 eV. What are the kinetic and potential energies of the electron in this state?

Ans:

Energy; $E = -\frac{ke^2}{2r}$

KE; $E_K = \frac{ke^2}{2r}$

PE, $E_P = -\frac{ke^2}{r}$

clearly $E_K = -E$

and $E_P = 2E$.

~~So~~, Now: $E = -13.6\text{ eV}$ (given)

So, $E_K = -(-13.6\text{ eV})$

$\boxed{E_K = 13.6\text{ eV}}$

and $E_P = 2(-13.6\text{ eV})$

$\boxed{E_P = -27.2\text{ eV}}$

Q.15. The wavelength of the second line of the Balmer series in the hydrogen spectrum is 4861 A°. Calculate the wavelength of the first lin

Ans: The wavelengths λ_1 and λ_2 of the first and second line of the Balmer series are given by

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$$

and, $$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}R$$

So, $$\frac{\lambda_1}{\lambda_2} = \frac{3}{16} \times \frac{36}{5} = \frac{27}{20}$$

$$\lambda_1 = \frac{27}{20} \times \lambda_2 = \frac{27}{20} \times 4861$$

$$\boxed{\lambda_1 = 6562\,A^\circ}$$

— 03 marks —

Q.16: What should be the minimum energy required by ground state electron in hydrogen atom so that the three lines are obtained in its emission spectrum?

Ans: To produce three lines, the number of excited energy levels can be obtained using the following relation

$$\frac{n(n-1)}{2} = 3$$

$$n^2 - n = 6$$

$$n^2 - n - 6 = 0$$

$$(n-3)(n+2) = 0$$

So, $n = 3$ or $n = -2$

Thus final minimum energy of electron will be

$$E_n = \frac{-13.6}{(3)^2}\,eV = \boxed{-1.5\,eV}$$

* Integral of two functions = I function x Integral of II function – integral of [differential coeff. of I x integral of II]

Q.17. Derive an expression for average life of a radioactive nuclide. Give its relationship with half life.

Ans: Consider a radionuclide which is having N_0 no. of atoms at $t=0$ and left after time 't' is N. Let after time $t+dt$ atoms reduces to $N-dN$, i.e in small time dt, the no. of atoms decayed is dN.

The life of each dN atoms $= t$

So, the total life of all dN atoms $= t \cdot dN$

Thus, total life of all the atoms in sample

$$= \int_0^{N_0} t\, dN$$

Hence average life

$$\tau = \frac{\int_0^{N_0} t\, dN}{N_0} \quad \text{—— (1)}$$

But $N = N_0 e^{-\lambda t}$.

$$\frac{dN}{dt} = -\lambda (N_0 e^{-\lambda t})$$

$$dN = -\lambda (N_0 e^{-\lambda t})\, dt$$

Putting this in (1), and when $N=0$, $t=\infty$ and $N=N_0$, $t=0$

$$\tau = \frac{\int_\infty^0 -\lambda N_0 e^{-\lambda t} dt \times t}{N_0}$$

$$\tau = \lambda \int_0^\infty t e^{-\lambda t} dt$$

*Integrating by parts we get

$$\tau = \lambda \left[\left| \frac{t e^{-\lambda t}}{-\lambda} \right|_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= \left| \frac{e^{-\lambda t}}{-\lambda} \right|_0^\infty = \frac{1}{\lambda}\left[e^{-\infty} - e^0\right] = \frac{1}{\lambda}[0-1]$$

$$\boxed{\tau = \frac{1}{\lambda}}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \boxed{0.693\tau}$$

$$\boxed{\tau = 1.44\, T_{1/2}}$$

Q.18: State Bohr's postulate for the permitted orbits for electron in a hydrogen atom. Use this postulate to prove that the circumference of the n^{th} permitted orbit for the electron can contain exact 'n' wavelengths of the de Broglie wavelength associated with the electron in that orbit.

Ans: According to Bohr:—
Electron can revolve only in that orbit for which its angular momentum is an integral multiple of $\frac{h}{2\pi}$. Where 'h' is Planck's constant i.e.

$$mvr = n\frac{h}{2\pi} \quad \text{—— (1)}$$

where $n = 1, 2, 3, ---$

We know that from de-Broglie hypothesis wavelength ~~associatat~~ associated with electron is —

$$\lambda = \frac{h}{mv}$$

or, $$mv = \frac{h}{\lambda} \quad \text{—— (11)}$$

putting (11) in (1)

$$\frac{h}{\lambda} r = n\frac{h}{2\pi}.$$

$$\boxed{2\pi r = n\lambda}$$

Thus the circumference $(2\pi r)$ of n^{th} permitted orbit of the electron can contain exactly 'n' wavelength of de Broglie wavelength associated with electron in that orbit.

Q.19. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released. Also calculate the energy released in MeV in the deutrium-tritium fusion reaction — $_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$

using data -

$m[{}_1H^2] = 2.014102\ u$

$m[{}_1H^3] = 3.016049\ u$

$m[{}_2He^4] = 4.002603\ u$

$m_n = 1.008665\ u$, $1u = 931.5\ \frac{MeV}{c^2}$

Ans: Nuclear fission is the phenomenon of splitting of a heavy nucleus (A>230) into two or more lighter nuclei –

$${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q.$$

Here $Q \sim 200.4$ MeV

Nuclear fusion is the phenomenon of fusion of two or more lighter nuclei to form a single heavy nucleus.

$${}_1H^1 + {}_1H^1 \longrightarrow {}_1He^2 + e^+ + \nu + 0.42\ MeV.$$

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1He^3 + n + 3.27\ MeV.$$

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1He^3 + {}_1H^1 + 4.03\ MeV$$

$${}_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + n\ MeV$$

Actually the mass the product nucleus is slightly ~~more than~~ less than the sum of the masses of the lighter nuclei fusing together. This is called mass defect. Acc-ording to relation $E = \Delta m c^2$ (J) it releases tremend-ous amount of energy.

Here $\Delta m = [(2.014102 + 3.016049) - (4.002603) + (1.008665)]\ u$

$\Delta m = 0.018883\ u.$

So Energy released

$Q = 0.18883 \times 931.5$ MeV

$Q = 17.589$ MeV.

Q.20: Prove that the radius of the n^{th} Bohr orbit of an atom is directly proportional to n^2, where 'n' is principal quantum number.

Q.

Ans: According to the Bohr's first postulate an electron in an atom could revolve in certain stable orbits without the emission of radiant energy. To keep the electron in its orbit the centripetal force on the electron must be equal to the electrostatic attraction between the nucleus and the electron. Therefore

$$\frac{mv^2}{r} = k\frac{Ze^2}{r} \qquad k = \frac{1}{4\pi\epsilon_0}$$

$$\text{So, } r = k\frac{Ze^2}{mv^2} \qquad \text{------ (1)}$$

where 'm' mass of electron, ~~...~~ v speed of revolution of electron in an orbit of radius 'r', $Z \rightarrow$ atomic no.

Acc. to second postulates of Bohr i.e. the electron revolves around nucleus in only those orbits for which its angular momentum is an integral multiple of $h/2\pi$ i.e.

$$mvr = n\frac{h}{2\pi}$$

$$\text{So, } r = \frac{nh}{2\pi mv} \qquad \text{------ (2)}$$

comparing 1 and 2

$$\frac{kZe^2}{mv^2} = \frac{nh}{2\pi mv}$$

$$v = \frac{2\pi kZe^2}{nh} \qquad \text{------ (3)}$$

Putting 3 in 1, we get

$$\boxed{r = \frac{n^2h^2}{4\pi^2 mkZe^2}}$$

Thus $\boxed{r \propto n^2}$