Topic:- Vector and 3D

One marks Question.

Q1 Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 4 respectively and $\vec{a}\cdot\vec{b} = 2\sqrt{3}$

Sol: Here $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2\sqrt{3}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{2\sqrt{3}}{\sqrt{3}\times 4}$$

$$= \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

Q2. If a vector makes α, β and γ angles from OX, OY, OZ axes respectively, Then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Sol: Let the d.c.'s of vectors are l, m and n

$\therefore$ $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$

Now, $l^2 + m^2 + n^2 = 1$.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1+1+1-1$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Q3: Find the value of λ, when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 unit.

Sol: we have $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$, $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(\lambda\hat{i} + \hat{j} + 4\hat{k})(2\hat{i} + 6\hat{j} + 3\hat{k})}{|2\hat{i} + 6\hat{j} + 3\hat{k}|}$$

$$= \frac{2\lambda + 6 + 12}{\sqrt{4+36+9}} = \frac{2\lambda + 18}{7}$$

$$\therefore \frac{2\lambda+18}{7} = 4 \Rightarrow 2\lambda = 10$$

$$\boxed{\lambda = 5}$$

Q4. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

Sol: Since $\vec{a} \parallel \vec{b} \Rightarrow \vec{a} = \lambda\vec{b}$

$$\Rightarrow 3\hat{i} + 2\hat{j} + 9\hat{k} = \lambda(\hat{i} + p\hat{j} + 3\hat{k})$$

on comparing, we get

$\lambda = 3$, $\lambda p = 2$, $3\lambda = 9$

$$p = \frac{2}{\lambda} = \frac{2}{3}.$$

Q5. What is the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis ?

Sol: Let $\vec{a} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{\sqrt{2+1+1}} = \frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$$

$$\therefore \cos\alpha = \frac{1}{\sqrt{2}}, \ \cos\beta = \frac{1}{2}, \ \cos\gamma = \frac{1}{2}$$

$\therefore$ The cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis is

$$\cos\beta = \frac{1}{2} = \cos\pi/3$$

$\therefore$ Required angle $= \frac{\pi}{3}$

2 Marks Question :

Q6. Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = 2\hat{i}-3\hat{j}-4\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$.

Sol: Given, $\vec{a}=2\hat{i}-3\hat{j}-4\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$

Then $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 2 & -3 & -4 \\ 1 & 2 & -1 \\ 3 & 1 & 2 \end{vmatrix}$

$= 2(4+1)+3(2+3)-4(1-6)$

Required Volume $= 45$

Q7. Show that the vectors $\vec{a}=3\hat{i}-4\hat{j}+5\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-3\hat{k}$ are coplanar.

Sol: Given $\vec{a}=3\hat{i}-4\hat{j}+5\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-3\hat{k}$

Then $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 3 & -4 & 5 \\ 2 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix}$

$= 3(3-2)+4(-6-1)+5(4+1)$

$= 0$

Hence, given vector are coplanar

Q8. Find the value of λ so that the planes $2x+\lambda y+3z=15$ and $x-y+7\lambda z=13$ are perpendicular to each other.

Sol: Since two planes are perpendicular to each other

$\therefore \quad a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow \quad 2(1)+\lambda(-1)+3(7\lambda)=0$

$\Rightarrow \quad 20\lambda=-2$

$\Rightarrow \quad \lambda=\frac{-1}{10}$

Q9. Find the distance between the planes $2x-y+3z-4=0$ and $6x-3y+9z+13=0$.

Sol: On putting $(0,0,0)$ in both the equation of planes, we get the opposite signs in L.H.S. of both side.

So, the planes are on the opposite side of the margin.

Distance between two parallel planes $= |OM| + |ON|$

$$= \frac{|2(0)+(-1)0+3(0)-4)|}{\sqrt{(2)^2+(-1)^2+(3)^2}} + \frac{|6(0)+-3(0)+0+13|}{\sqrt{(6)^2+(-3)^2+(9)^2}}$$

$$= \frac{4}{\sqrt{14}} + \frac{13}{3\sqrt{14}}$$

$$= \frac{25}{3\sqrt{14}} \text{ units.}$$

Q10. Find the angle between $\vec{r} = -\hat{i} + \lambda(3\hat{i}-\hat{j}+4\hat{k})$ and $\vec{r}\cdot(2\hat{i}-4\hat{j}+\hat{k}) - 5 = 0$

Sol: First equation is the equation of straight line and the other one is the equation of plane.

So, required angle is

$$\sin\theta = \frac{3(2)+(-1)(-4)+4(1)}{\sqrt{9+1+16}\cdot\sqrt{4+16+1}}$$

$$= \frac{6+4+4}{\sqrt{26}\sqrt{21}}$$

$$= \frac{14}{\sqrt{546}}$$

$$\theta = \sin^{-1}\left(\frac{14}{\sqrt{546}}\right)$$

4 Marks Question

Q11. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, prove that
$$[\vec{a} \quad \vec{b}+\vec{c} \quad \vec{a}+\vec{b}+\vec{c}] = 0$$

Sol: $[\vec{a} \quad \vec{b}+\vec{c} \quad \vec{a}+\vec{b}+\vec{c}]$

$= [\vec{a}\times(\vec{b}+\vec{c})] \cdot [\vec{a}+\vec{b}+\vec{c}]$

$= [(\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})] \cdot [\vec{a}+\vec{b}+\vec{c}]$ (Distributive law)

$= (\vec{a}\times\vec{b})\cdot\vec{a} + (\vec{a}\times\vec{b})\cdot\vec{b} + (\vec{a}\times\vec{b})\cdot\vec{c} + (\vec{a}\times\vec{c})\cdot\vec{a}$
$\quad + (\vec{a}\times\vec{c})\cdot\vec{b} + (\vec{a}\times\vec{c})\cdot\vec{c}$ (Distributive law)

$= [\vec{a}\,\vec{b}\,\vec{a}] + [\vec{a}\,\vec{b}\,\vec{b}] + [\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{c}\,\vec{a}]$
$\quad + [\vec{a}\,\vec{c}\,\vec{b}] + [\vec{a}\,\vec{c}\,\vec{c}]$

$= [\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{c}\,\vec{b}]$

$= [\vec{a}\,\vec{b}\,\vec{c}] - [\vec{a}\,\vec{b}\,\vec{c}] = 0$

Hence, $[\vec{a} \quad \vec{b}+\vec{c} \quad \vec{a}+\vec{b}+\vec{c}] = 0$

Q12. Show that the points $A(-1,4,-3)$, $B(3,2,-5)$, $C(-3,8,-5)$ and $D(-3,2,1)$ are coplanar.

Sol: Clearly, the position vectors of A, B, C, D are $(-\hat{i}+4\hat{j}-3\hat{k})$, $(3\hat{i}+2\hat{j}+\hat{k})$, $(-3\hat{i}+8\hat{j}-5\hat{k})$, $(3\hat{i}+2\hat{j}-5\hat{k})$

$\vec{AB} = (\text{P.V of } B) - (\text{P.V of } A)$
$= (3\hat{i}+2\hat{j}-5\hat{k}) - (-\hat{i}+4\hat{j}-3\hat{k})$
$= (4\hat{i}-2\hat{j}-2\hat{k})$

$\vec{AC} = (\text{P.V of } C) - (\text{P.V. of } A)$
$= (-3\hat{i}+8\hat{j}-5\hat{k}) - (-\hat{i}+4\hat{j}-3\hat{k})$
$= -2\hat{i}+4\hat{j}-2\hat{k}$

$\vec{AD} = (\text{P.V of } D) - (\text{P.V of } A)$
$= (-3\hat{i}+2\hat{j}+\hat{k}) - (-\hat{i}+4\hat{j}-3\hat{k})$
$= -2\hat{i}+4\hat{j}-2\hat{k}$

$$[\vec{AB}\ \vec{AC}\ \vec{AD}] = \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix}$$

$$= 2\times(-2)\times(-2) \begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 8 \begin{vmatrix} 0 & -3 & 3 \\ 0 & -3 & 3 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 8\times 0 = 0$$

$\vec{AB}$ $\vec{AC}$ $\vec{AD}$ are coplanar

The points A, B, C, D are Coplanar.

Q13. Find the coordinates of the foot of the perpendicular drawn from the point A (1, 2, 1) to the line joining B (1, 4, 6) and C (5, 4, 4)

Sol: Drawn $AN \perp BC$

Let N divide BC in the ratio K : 1

Then, the Coordinates of N are

$$\left(\frac{5K+1}{K+1}, \frac{4K+4}{K+1}, \frac{4K+6}{K+1}\right)$$

d.r's of AN are

$$\left(\frac{5K+1}{K+1}-1, \frac{4K+4}{K+1}-2, \frac{4K+6}{K+1}-1\right)$$

$$\left(\frac{4K}{K+1}, \frac{2K+2}{K+1}, \frac{3K+5}{K+1}\right)$$

And the d.r's of BC are $(5-1), (4-4), (4-6)$

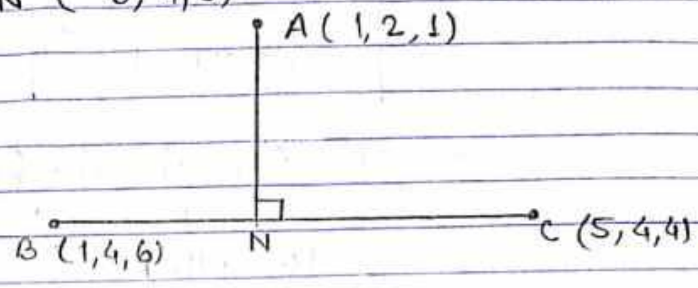
i.e. $4, 0, -2$

Since $AN \perp BC$, we have

$$\left(4\times\frac{4K}{K+1}\right) + 0\times\left(\frac{2K+2}{K+1}\right) - 2\times\left(\frac{3K+5}{K+1}\right) = 0$$

$$16K + 0 - 6K - 10 = 0$$
$$10K = 10$$
$$K = 1$$

Putting $K = 1$, we get the coordinates of N $(3, 4, 5)$



Q14. Find the shortest distance between the lines whose vectors equations are:

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}, \text{ and}$$
$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}.$$

Sol: The given equation can be written as

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k}) \text{ and}$$
$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$$

Comparing the given equation with the standard equation

$\vec{r} = \vec{a_1} + t\vec{b_1}$ and $\vec{r} = \vec{a_2} + s\vec{b_2}$, we get

$$\vec{a_1} = (\hat{i} - 2\hat{j} + 3\hat{k}), \quad \vec{b_1} = (-\hat{i} + \hat{j} - 2\hat{k})$$
$$\vec{a_2} = (\hat{i} - \hat{j} - \hat{k}) \quad \vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$
$$= (\hat{j} - 4\hat{k})$$

$$\text{and } (\vec{b_1} + \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= (-2+4)\hat{i} - (2+2)\hat{j} + (-2-1)\hat{k}$$
$$= (2\hat{i} - 4\hat{j} - 3\hat{k})$$

$$|\vec{b_1}+\vec{b_2}| = \sqrt{2^2+(-4)^2+(-3)^2}$$
$$= \sqrt{4+16+9}$$
$$= \sqrt{29}$$

$$\therefore \quad SD = \left|\frac{(\vec{a_2}-\vec{a_1})\cdot\vec{b_1}\times\vec{b_2}}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$= \frac{|(\hat{j}-4\hat{k})\cdot(2\hat{i}-4\hat{j}-3\hat{k})|}{\sqrt{29}}$$

$$= \frac{|0-4+12|}{\sqrt{29}}$$

$$= \frac{8\sqrt{29}}{29} \text{ unit.}$$

Q15. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other. Find their point of intersection.

Sol: The equation of given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \quad \text{—(i)}$$

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu \quad \text{—(ii)}$$

any point on the line (i) is $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$
any point on the line (ii) is $Q(5\mu+4, 2\mu+1, \mu)$
If, the lines intersects then P and Q must coincide for some particular values of λ and μ.
This gives

$$2\lambda+1 = 5\mu+4, \; 3\lambda+2 = 2\mu+1 \text{ and } 4\lambda+3 = \mu$$

$$2\lambda - 5\mu = 3 \quad \text{—(i)}$$
$$3\lambda - 2\mu = -1 \quad \text{—(ii)}$$
$$4\lambda - \mu = -3 \quad \text{—(iii)}$$

on solving (i) and (ii), we get $\lambda = -1$ and $\mu = -1$

These values of λ and μ also satisfy (iii)
Hence, the given line intersects
putting $\lambda = -1$, we get $P(-1, -1, -1)$
putting value of $\mu = -1$, we get $Q(-1, -1, -1)$

Hence the point of intersection of the given lines is $(-1, -1, -1)$

6 Marks Question

Q16. Find the vector and cartesian equation of the line passing through the points $(1, 2, -4)$ and perpendicular to each of the lines.
$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x+15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$$

Sol: The given lines are
$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ —(i) and}$$
$$\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5} \text{ — (ii)}$$

Let a, b, c be the direction ratios of the required line.
Then, it being perpendicular to each ~~other~~ of the lines (i) and (ii), we have
$$3a - 16b + 7c = 0 \text{ and}$$
$$3a + 8b - 5c = 0$$

on solving these equation by cross multiplication we get

$$\frac{a}{80-56} = \frac{b}{(21+15)} = \frac{c}{(24+48)}$$

$$\frac{a}{24} = \frac{b}{36} = \frac{c}{72}$$

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$

Thus the direction ratios 2, 3, 6.

The equation of the line passing through the point A (1, 2, −4) and having 2, 3, 6 as its direction ratios.

Equation in cartesian form are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

position vector of point A is $\vec{r_1} = (\hat{i} + 2\hat{j} - 4\hat{k})$

Also, The required line has direction ratios 2, 3, 6 and so it is parallel to Vector $\vec{m} = (2\hat{i} + 3\hat{j} + 6\hat{k})$

The equation in vector form is

$$\vec{r} = \vec{r_1} + \lambda \vec{m}$$

$$= (\hat{i}+2\hat{j}-4\hat{k}) + \lambda\,(2\hat{i}+3\hat{j}+6\hat{k})$$

Q17. Find the image of the point P(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Sol. The equation of the given line are

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = r \text{ (say)}$$

The general point on this line is

$(r,\ 2r+1,\ 3r+2)$

Let N be the foot of perpendicular drawn from the point $P(1,6,3)$ on the given line.

Then, this point is $N(r, 2r+1, 3r+2)$ for some fixed value of r.

D.r's of PN are $(r-1, 2r-5, 3r-1)$
D.r's of the given line are 1, 2, 3

Sine PN is perpendicular to given-line (i)

$1\cdot(r-1) + 2(2r-5) + 3(3r-1) = 0$

$14r = 14$

$r = 1$

So, the point N is given by $N(1,3,5)$
Let $Q(\alpha, \beta, r)$ be the image of $P(1,6,3)$ in the given line

Then, N is the midpoint of PQ

$\frac{\alpha+1}{2} = \frac{\beta+6}{2} = \frac{r+3}{2} = 5$

$\alpha = 1, \quad \beta = 0, \quad \text{and } r = 7$

Hence, the image of the point $P(1,6,3)$ in the given line is $Q(1,0,7)$

Q18. A variable plane is at a constant distance p from the origin and meets the coordinate axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is

$x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$

Sol: Let the equation of the variable plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{——} \quad (i)$

this plane meets the x axis, y-axis and z-axis at the point $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$ respectively.

Let (α, β, γ) be the coordinates of the centroid of the tetrahedron OABC

then $\alpha = \frac{0+a+0+0}{4} = \frac{a}{4}$

$\beta = \frac{0+0+b+0}{4} = \frac{b}{4}$ and

$\gamma = \frac{0+0+0+c}{4} = \frac{c}{4}$

$a = 4\alpha, \quad b = 4\beta, \quad c = 4\gamma$

P = distance of the plane (i) from $(0, 0, 0)$.

$$P = \frac{\left|\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1\right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\frac{1}{P} = \frac{1}{p^2} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$= \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)$$

$$\frac{1}{P^2} = \frac{1}{16\alpha^2} + \frac{1}{16\beta^2} + \frac{1}{16\gamma^2} \quad \text{(using ii)}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{16}{P^2}$$

$$\alpha^{-2} + \beta^{-2} + \gamma^{-2} = 16P^{-2}$$

Hence the required locus is

$$x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$$

Q19. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$

Sol: Let the required equation of the plane be

$$(2x+y-z-3)+\lambda(5x-3y+4z+9)=0$$

$$(2+5\lambda)x+(1-3\lambda)y+(-1+4\lambda)z-3+9\lambda=0$$

the plane given by (i) is parallel to the line

$$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$$

So, the normal to the plane (i) is perpendicular to the line (i)

$$2(2+5\lambda)+4(1-3\lambda)+5(-1+4\lambda)=0$$

$$18\lambda=-3$$

$$\lambda=\frac{-3}{18}=\frac{-1}{6}$$

putting $\lambda=\frac{-1}{6}$ in (i), we get

$$\left(2-\frac{5}{6}\right)x+\left(1+\frac{3}{6}\right)y+\left(-1-\frac{4}{6}z\right)-9\times\frac{1}{6}=0$$

$$\frac{7x}{6}+\frac{9y}{6}-\frac{10z}{6}-\frac{27}{6}=0$$

$$7x+9y-10z-27=0$$

Hence the required equation of the plane is $7x+9y-10z-27=0$

Q20. Find the equation of the plane passing through the point $A(1,2,1)$ and perpendicular to the the line joining the points $P(1,4,2)$ and $Q(2,3,5)$ also find the distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$

Sol: The general equation of a plane passing through the point A (1, 2, 1) is given by

$$a(x-1)+b(y-2)+c(z-1)=0 \quad \text{—(i)}$$

Dir.'s of normal to the plane (i) are a, b, c

Dir's of the line PQ are (2, -1) (3, -4) (5, -2) ie 1, -1, -3.

Required plane is perpendicular to line PQ, The normal of this plane must parallel to PQ

$$\frac{a}{1}=\frac{b}{-1}=\frac{c}{3}=\lambda$$

$$a=\lambda, \ b=-\lambda \text{ and } c=3\lambda$$

putting the values in equation (i)

$$\lambda(x-1)+\lambda(y-2)+3\lambda(z-1)=0$$

$$(x-1)-(y-2)+3(z-1)=0$$

$$x-y+3z=2 \quad \text{—(ii)}$$

Hence the required equation of the plane is $x-y+3z=2$

the given line is $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$ —(iii)

This line passes through the point (-3, 5, 7)

Distance between the plane (ii) and the line (iii)

= Distance of any point on this line from the plane.

= length of perpendicular from (-3, 5, 7) on the plane (ii)

$$= \frac{|-3-5+21-2|}{\sqrt{1+1+9}}$$

$$= \frac{11}{\sqrt{11}} = 11 \text{ units}$$

Hence, the distance between the desired plane and the given line is $\sqrt{11}$ units.