Linear Programming

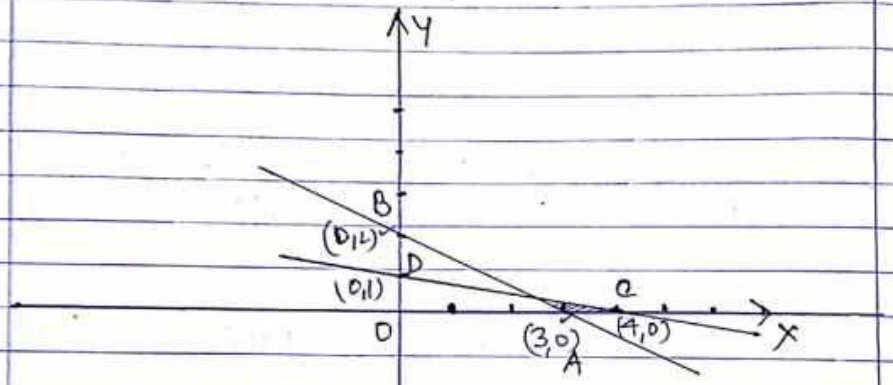
Question 1: Draw the graph of the Solution Set of the System of inequalities

$2x+3y \geq 6,\ x+4y \leq 4,\ x \geq 0,\ y \geq 0$

Sol^n:→ Consider the equations

$2x+3y=6,\ x+4y=4,\ x=0,\ y=0$

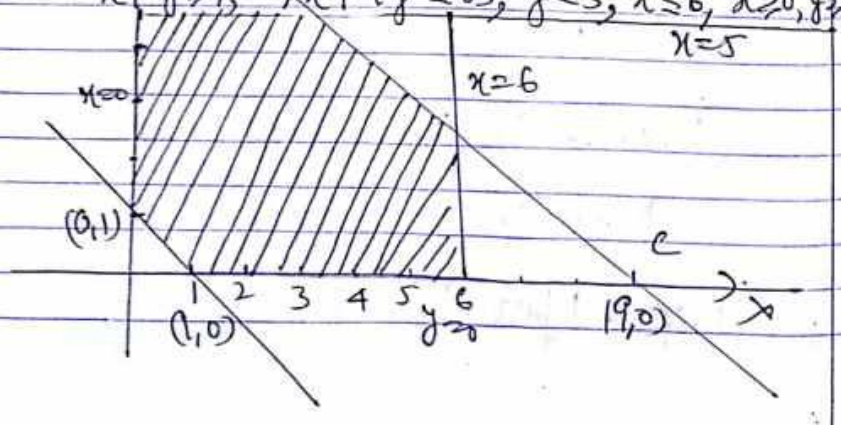
$\frac{x}{3}+\frac{y}{2}=1,\ \frac{x}{4}+\frac{y}{1}=1,$



Q.2. Exhibit graphically the Solution Set of the System of linear inequalities

$x+y \geq 1,\ 7x+9y \leq 63,\ y \leq 5,\ x \leq 6,\ x \geq 0, y \geq 0.$

Sol.

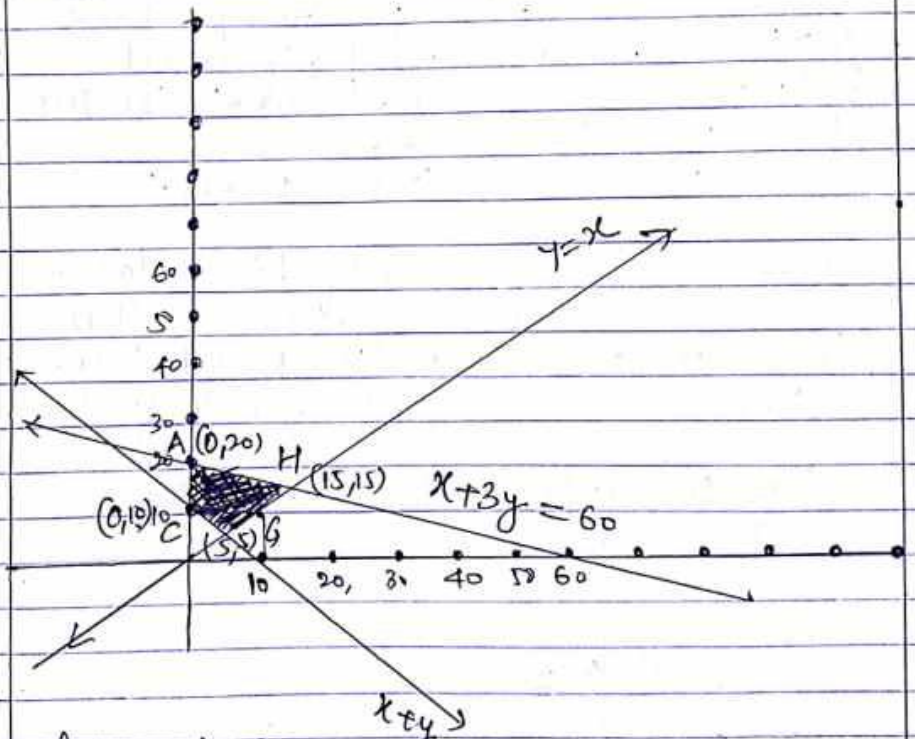


Question 3. Solve the following Problem graphically

Minimize and maximize $Z = 3x + 9y$, Subject to the Constraints

$x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x \geq 0$, and $y \geq 0$

Solution



Value of $Z = 3x + 9y$

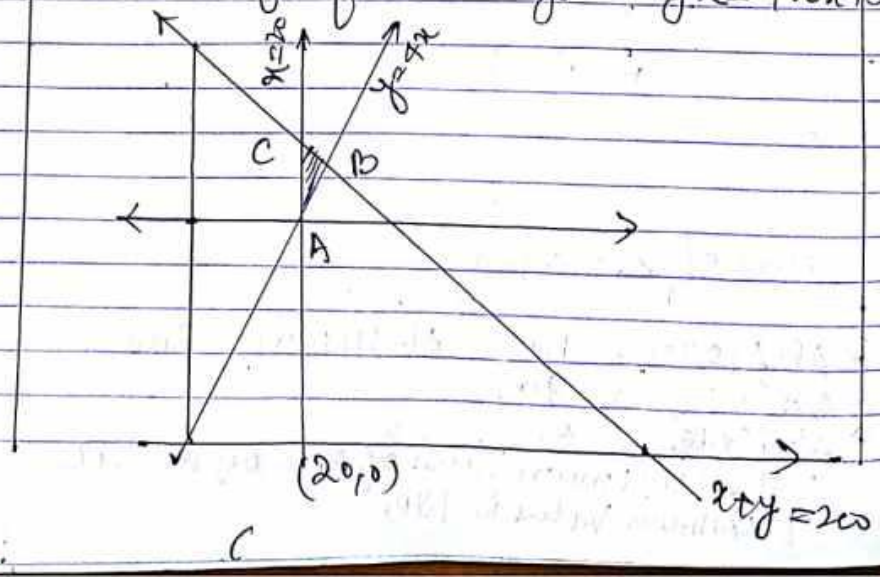
- At A(0,20), = 180 At H(15,15), 180
- At C(0,10), = 90
- At G(5,5) = 60

So the Minimum Value of Z is 60, and its Maximum Value is 180,

Q4. An aeroplane of an airline can carry a maximum of 200 passengers. A profit of ₹ 400 is made on each first class ticket and a profit of ₹ 300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However at least 4 times as many passengers prefer to travel by economy class then by first class. Determine how many of each type of ticket must be sold in order to maximize the profit for the airline.
What is the maximum profit.?

Solution Let x tickets of first class and y ticket of economy class be sold to maximize the profit Then.
$x \geq 20$, $y \geq 4x$, $y \geq 80$, and $x + y \leq 200$

The profit function is given by $Z = 400x + 300y$



Coordinate A(20,80),

$Z = 400x + 300y$

$Z = 400 \times 20 + 300 \times 80$

$= 8000 + 24000$

$= 32000$

B(40,160), $Z = 400x + 300y$

$= 400 \times 40 + 300 \times 160$

$= 64000$

C(20,180), $Z = 400x + 300y$

$= 400 \times 20 + 300 \times 180$

$= 62000$

$\therefore Z$ is maximum at $x = 40$, $y = 160$

Question 5

A Company makes two types of belts A and B, Profits on these belts being ₹4 and ₹3 each respectively. Each belt of type A requires twice as much time as a belt of type B. and if all belts were of type B, the Company could make 1000 belts per day the supply of leather is sufficient for only 800 belts per day (both A and B) Combined). At the most 400 buckles for belts of type A and 700 for those of type B. are available per day. How many belts of each type should the Company make per day so as to maximize the Profit?

6. Solution:-

Let x belts of types A and y belts of type B be made then

$x \geq 0$, $y \geq 0$, $x \leq 400$, $y \leq 700$ and $x + y \leq 800$

Also 1000 belts of type B can be made in 1 day

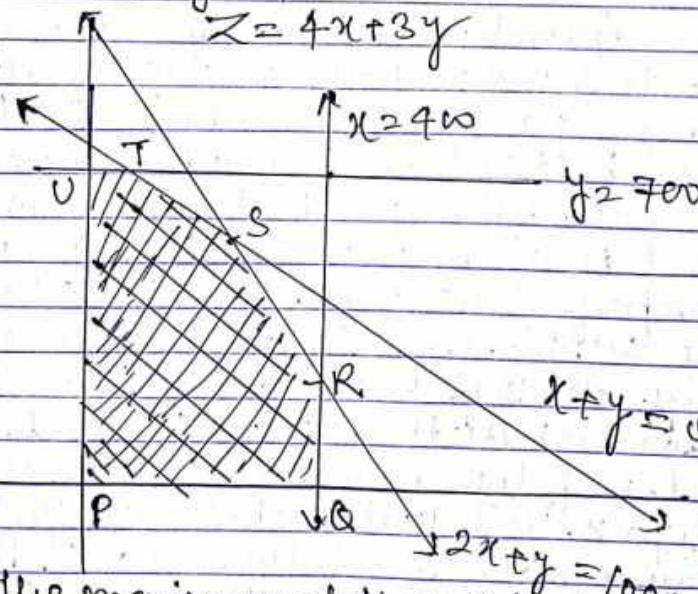
500 belt of type A can be made in 1 day

$\therefore$ time taken to make x bells of type A and y bells of types B.

$$= \left(\frac{x}{500} + \frac{y}{1000}\right) \text{ days}$$

$2x + y \leq 1000$

$Z = 4x + 3y$



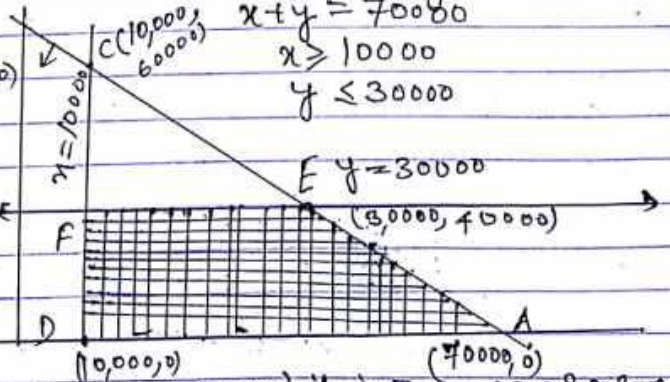
The maximum of these values is 2600 occurring at (200, 600).

$\therefore$ Z. is maximum when $x = 200$ and $y = 600$.

Q 6. A retired person has ₹ 70000 to invest and two types of bonds are available in the market for investment. First type of bond yield an annual income of 8% on the amount invested and the second type of ~~bond~~ bond yield 10% per annum. As per norms ~~two~~ he has to invest minimum of ₹ 10000 in the first type and not more than ₹ 30000 in the second type. How should he plan his investment so as to get maximum return after one year of investment?

Solution Let bonds A be at 8% and bonds B be at 10%. Suppose he plans to invest ₹ x in bonds A and ₹ y in bonds B.
Then clearly $x + y = 70000$

He invests minimum of ₹ 10000 in bonds A
$x \geq 10000$
Also he invests not more than ₹ 30000 in bonds B. $y \leq 30000$
Let $\boxed{Z = 0.08x + 0.1y}$



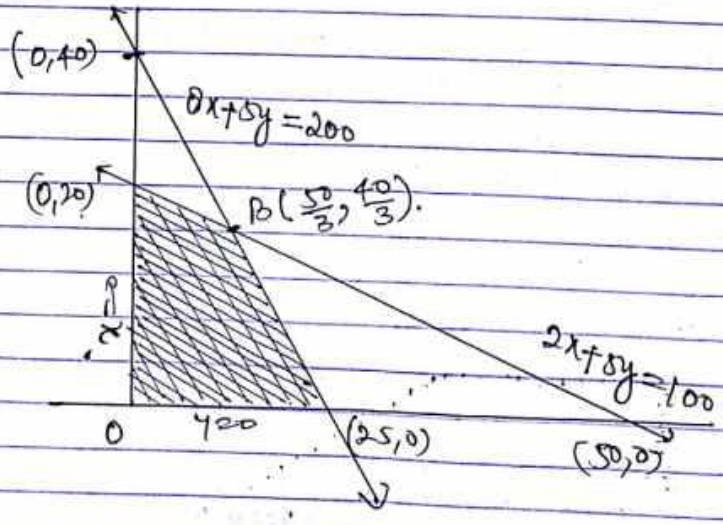
At E(30000, 40000) it is $Z = (0.08 \times 30000 + 0.1 \times 40000)$
So in order to get a maximum annual return = 6400
he should invest ₹ 30000 in bond A and ₹ 40000 in bond B.

Q.7, If a young man rides his motorcycle at 25 km per hour, he has to spend ₹2 Kilometer on petrol. If he rides it at a faster speed of 40 km per hours, the petrol cost increases to ₹5 per Kilometre. He has ₹100 to spend on petrol and wishes to find the maximum distance he can travel within one hours. Express this as a Linear Programming Problem and then solve it.

Solution: Suppose that the young man rides x km at 25 km per hour and y km at 40 km per hour. Then we have to maximize $P = x + y$

Clearly $x \geq 0$, $y \geq 0$, $2x + 5y \leq 100$

$\frac{x}{25} + \frac{y}{40} \leq 1$, $8x + 5y \leq 200$



$\frac{x}{50}+\frac{y}{20}=1$, $\frac{x}{25}+\frac{y}{40}=1$,

$P=x+y$ at these Points (0, 25, 30, and 20 respectly)

So $P=x+y$ is maximum when $x=\frac{50}{3}$ & $y=\frac{40}{3}$,

Thus the young man can cover the maximum distance of 30km if he rides $\frac{50}{3}$ km at 25 km/h and $\frac{40}{3}$ km at 40 km/h.