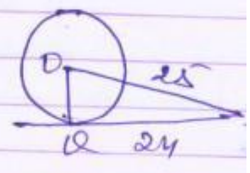
CIRCLES.

Q1. The length of tangent to a circle from P, which is 25cm away from centre is 24cm. What is the radius of circle.

Soln:



$OQ = \sqrt{OP^2 - QP^2}$

$= \sqrt{25^2 - 24^2}$

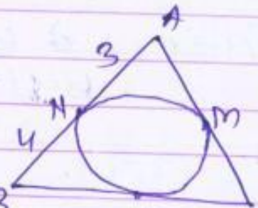
$= \sqrt{625 - 576}$

$= \sqrt{49}$

$\boxed{OQ = 7\ cm}$ Ans

Q2. Find length of BC.

AN=3, BN=4, AC=9



Soln.

BN=BL (Tangents from ext. point)

BL = 4 — (I)

Similarly AN = AM = 3

and CM = AC − AM

CM = 9 − 3 = 6

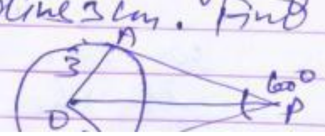
again, CL = CM = 6 — (II)

So, BC = BL + CL

= 4 + 6

$\boxed{BC = 10\ cm}$ Ans

Q3. If two tangents inclined at an angle of 60° are drawn to circle of radius 3cm. Find length of tangents.



Sol^n: $\angle APO = 30^\circ$

$\therefore$ In $\triangle AOP$

$\tan 30 = \frac{OA}{AP}$

$\frac{1}{\sqrt{3}} = \frac{3}{AP}$

$\boxed{AP = 3\sqrt{3}\,cm}$ Ans

Q4. Prove that the length of tangen[t] from an external point are eq[ual]

Given: a circle with centre O.

To Prove: $AP = AQ$

Pf: In $\triangle OPA$ and $\triangle OQA$

$OP = OQ$ (radii)

$\angle P = \angle Q$ 90°

$OA = OA$ common

$\triangle OPA \cong \triangle OQA$ (RHS)

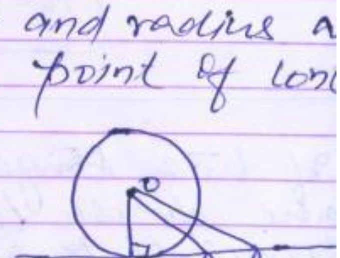
$\therefore \boxed{AP = AQ \text{ (CPCT)}}$ Proved

Q5 Prove that tangent and radius a[re] perpendicular at point of con[tact]

Pf $\rightarrow$ $\because OP_1 > OP$

$OP_2 > OP$

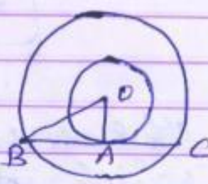
we can say OP is shortest distance



So, $OP \perp AB$

Prd.

Q6. If radii of two concentric circles are 4 cm and 5 cm, then find length of each chord of one circle which is tangent to the other circle.



Solⁿ: $OA = 4, OB = 5$

$AB = \sqrt{OB^2 - OA^2}$

$AB = \sqrt{5^2 - 4^2}$

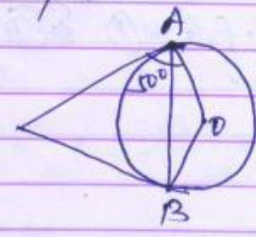
$AB = \sqrt{25 - 16}$

$AB = \sqrt{9}$

$\boxed{AB = 3\text{ cm}}$ Ans.

Q7. From point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^\circ$ then find $\angle AOB$.

Solⁿ



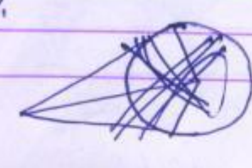
$\angle APB = 180 - 50 - 50$

$\angle APB = 180$

$\angle AOB = 180 - 80$

$\boxed{\angle AOB = 100^\circ}$

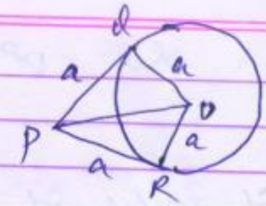
Q8. If the angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 90°, ~~of~~ find OP.



Sol^n

Let PQ, PR be tangents

$\therefore \angle P = 90^\circ, \angle QOR = 90^\circ$



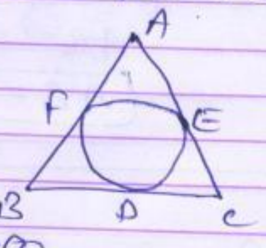
Also, OR = OQ = a

$\therefore$ PQOR is a square.

$\Rightarrow OP = \sqrt{a^2+a^2} = \sqrt{2a^2} = a\sqrt{2}$

$\boxed{OP = a\sqrt{2}}$ Ans.

Q9. The incircle of an isosceles $\triangle ABC$ in which AB = AC, touches the sides BC, and AB at D, E and F. Prove BD = DC



Given: AB = AC

To prove: BD = DC

Pf: $\because$ AB = AC

AF + BF = AE + CE —(I)

BF = BD & CE = CD (Tangents from external point)

AF = AE

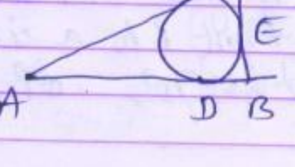
from (I) & (II)

$\cancel{AE} + BD = \cancel{AE} + CD$

$\boxed{BD = CD}$ Proved

Q10- In given fig. find AD, BE and CF.

AB = 12 cm
BC = 8 cm } given.
AC = 10 cm



∵ Tangents from ext. point are equal

Solⁿ: ∴ $AD = AF = x$

$DB = BE = 12 - x$

and $CF = CE = 10 - x$

∴ $BC = BE + EC$

$8 = 12 - x + 10 - x$

$8 - 22 = -2x$

$-14 = -2x$

$2x = 14$

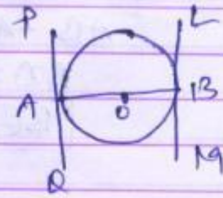
$x = 7$

$AD = 7$, $BE = 12 - 7 = 5$, $CF = 10 - 7 = 3$

$\boxed{AD = 7cm,\ BE = 5cm,\ CF = 3cm}$ Ans.

Q11. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Pf → ∵ Tangents and radius are ⊥ar to each other at point of contact.



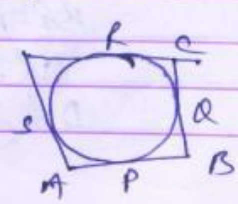
∴ $OA \perp PQ$ & $OB \perp LM$

∴ $\angle BAQ = \angle ABL = 90^\circ$ (alternate angles)

So, $\boxed{PQ \parallel LM}$ Proved

Q12. A Quadrilateral ABCD is drawn to circumscribe a circle. Prove $AB + CD = AD + BC$

Pf → ∵ tangents from ext. point are equal

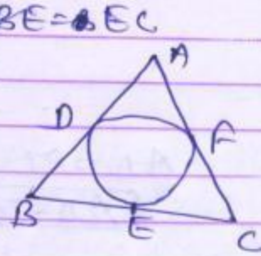


$\therefore$ AP = AS , BP = BQ, DR = DS , CR = CQ
Adding all these we get

$AP + BP + CR + DR = AS + DS + BQ + CQ$
$AB + CD = BC + DA$

proved

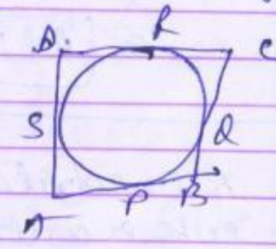
Q13. In fig. AB = AC prove BE = EC



$\because$ Tangents from external point are equal.

$\therefore$ $AD = AF$ — (i)
$BD = BE$ — (ii)
$CE = CF$ — (iii)
$AB = AC$
$AB - AD = AC - AD$
$AB - AD = AC - AF$
$BD = CF$
$\boxed{BE = CE}$ from (iii) proved

Q14. Prove that the ||gm circumscribing a circle is a rhombus.



Pf → $\because$ Tangents from ext. point are equal

$\therefore$
$AP = AS$ — (i)
$BP = BQ$ — (ii)
$CR = CQ$ — (iii)
$DR = DS$ — (iv)

(I) + (II) − (III) + (IV) we have:

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$

$AB + AB = BC + BC$ (ABCD is a llgm)

$2AB = 2BC$

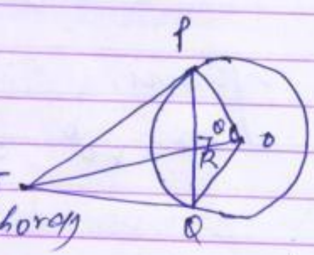
$AB = BC$

Similarly $AB = BC = CD = DA$

So ABCD is a rhombus. Proved

Q15. In fig. PQ is a chord of length 16cm, radius of circle 10cm. Find TP

given: $PQ = 16$, $PO = 10$



Soln: $PR = RQ = 8cm$

($\perp^{ar}$ from centre bisects chord)

$\therefore OR = \sqrt{OP^2 - PR^2} = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36}$

$\boxed{OR = 6cm}$

In rt Δ POR $\tan\theta = \frac{PR}{OR} = \frac{8}{6} = \frac{4}{3}$

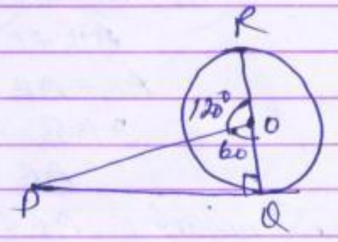
In rtΔOPT, $\tan\theta = \frac{OP}{TP}$

$\frac{4}{3} = \frac{10}{TP}$

$TP = \frac{10 \times 3}{4} = 7.5cm$

$\boxed{TP = 7.5cm}$ Ans

Q16. In fig. $\angle POR = 120^\circ$, diameter $QR = 8$ cm, find OP and PQ.



Soln: $OQ = OR = 4$ cm

$\angle POQ = 180 - 120$

$\angle POQ = 60^\circ$

Now in rt $\triangle POQ$

$$\cos 60^\circ = \frac{OQ}{OP}$$

$$\frac{1}{2} = \frac{OQ}{OP} \Rightarrow \frac{1}{2} = \frac{4}{OP}$$

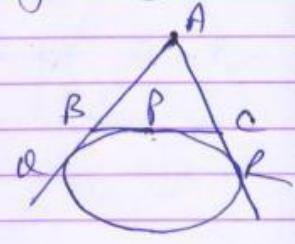
$$\therefore \boxed{OP = 8 \text{ cm}}$$

again,

$$\tan 60^\circ = \frac{PQ}{OQ} \Rightarrow \sqrt{3} = \frac{PQ}{4}$$

$$\boxed{PQ = 4\sqrt{3}\text{ cm}}$$ Ans

Q17. A circle is touching the side BC of $\triangle ABC$ at P and touching AB & AC produced at Q and R. Prove that
$AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC$)



Pf → ∵ Tangents from ext. point are equal.

$\therefore BP = BQ$ — (I)

$CP = CR$ — (II)

$AQ = AR$ — (III)

From (III) $AQ = AR$

$AB + BQ = AC + CR$

$AB + BP = AC + CP$ (from (I) & (II)) — (IV)

Now Perimeter of $\triangle ABC = AB + BC + CA$

$= AB + (BP + CP) + CA$

$= AB + BP + CP + CA$

$= (AB + BP) + (AB + BP)$.

$= 2(AB + BP)$ $\longrightarrow$ from (IV)

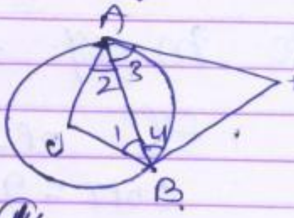
$= 2(AB + BQ)$ from (I)

$= 2\,AQ$

$\therefore AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC$)

Proved

Q10. Prove that the tangents drawn at ends of a ~~circle~~ chord make equal angles with the chord.



To Proves

$\angle PAB = \angle PBA$

Pf $\Rightarrow$ In $\triangle OAB$

$OA = OB \Rightarrow \angle 1 = \angle 2$ — (I)

$\angle 1 + \angle 4 = \angle 2 + \angle 3$ (II) (Both 90° as Radius ⊥ tangent)

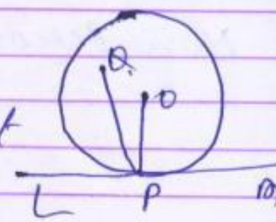
(II) − (I) $\cancel{\angle 1} + \angle 4 - \cancel{\angle 1} = \cancel{\angle 2} + \angle 3 - \cancel{\angle 2}$

$\angle 4 = \angle 3$

$\angle PAB = \angle PBA$ Proved.

Q19. Prove that the perpendicular at the point of contact to tangent to a circle passes through the centre.

Pf → Let $PQ \perp LM$



Now ∵ tangent at a point to a circle is $\perp^{ar}$ to the radius.

∴ $OP \perp LM \Rightarrow \angle OPM = 90^\circ$

Also, $\angle QPM = 90^\circ$

$\angle OPM = \angle QPM$

which is not possible, it is only when Q coincides at O.

Hence the $\perp^{ar}$ at the point of contact to tangent to a circle passes through centre. Proved

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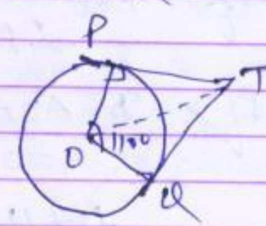
Q20. In fig. $\angle POQ = 110^\circ$ find $\angle PTQ$

Soln: $OP \perp PT$, $OQ \perp TQ$



$\angle OPT = 90^\circ$ $\angle OQT = 90^\circ$

$\angle POQ = 110^\circ$

In Quad. OPTQ, we have

$\angle POQ + \angle OPT + \angle PTQ + \angle TQO = 360$

$110 + 90 + \angle PTQ + 90 = 360$

$\angle PTQ = 360 - 290$

$\boxed{\angle PTQ = 70^\circ}$ Ans